Module 4: Equations and Inequalities

Slide 1: This is Module 4 Equations and Inequalities. We encourage you to follow along and construct the visual representations discussed within this session. Feel free to pause the presentation at any point.

Slide 3-10: The equal sign is a mathematical symbol that is often misunderstood by students.
Research shows most elementary students gave an answer of 12 or 17 when presented with this equation. Of the sixth graders observed during this research project, none provided the correct answer of 7.
What implications does understanding the equal sign have on students' understanding? It's important for students to understand and symbolize relationships in our number system. The equal sign is a key method of representing these relationships. Without this understanding, students typically have difficulty with algebraic expressions.
To reinforce this critical understanding, you can conceptualize the equations in this manner. As discussed in module 2, you would show an image like this to students and ask what do you notice? What do you wonder? The idea of balance and equality helps with understanding the meaning of the equal sign.
Try incorporating true/false sentences such as these. Students must provide justifications for their answer of true or false. Notice how these sentences require computation on different sides of the equal sign. Traditionally, the computation is shown on the left side of the equal sign.
Transition to equations where computation appears on both sides of the equal sign to further develop the equivalence meaning of the equal sign. Students continue to justify their thinking.
Relational thinking can be used when solving problems once students understand the equal sign means the quantities on both sides are the same. Relational thinking occurs when students notice and apply numeric relationships between the two sides instead of actually computing the amounts.

Slide 11-13: Variables can be used to represent: A unique but unknown quantity and A quantity that varies.
In this example, students work to identify the weight of each ball. Allow students time to explore and make observations that help them figure out other facts. Encourage students to use models to represent and explore the problems.
It's important for students to know variables can have many possible numeric solutions such as with functions. The number line is a good tool to help develop the concept of variable. Notice this example. Using this example, you could ask students what's the value of X? Could X be any value? What number could not be a value of X?

Slide 14-21: let's turn our attention to teaching strategies for solving equations.
Similarly, to using Cuisenaire rods as fraction length models, you can use Cuisenaire rods to model one step equations. Through a brief exploration of the rods, students will discover the orange rods represent 10 units and the blue which is a tad bit shorter than the orange represents 9. One white rod is one tenth of the orange and one half of the red. Therefore, the red rod is representing two. Looking carefully at

the model, we are comparing 39 to 22. The unknown is the amount it takes for the lengths to be equivalent.

We can reason that an additional ten can be added to twenty-two making the bottom length 32. From 32 seven more units would need to be added in order for the lengths to be equivalent. Another student maybe reason it this way, the equivalent amount in each length is 22. 39-22 is 17, therefore the unknown amount is seventeen. It is important to continually reinforce the idea of equivalence as you discuss the concrete problem and the algebraic equation.

Bar diagrams provide another visual of equivalent expressions when setting up equations. In this diagram we are comparing three times a number plus 5 which is the same as 38. Written algebraically as $3x + 5 = 38$, students would solve for x. With the bar diagram students could begin thinking through how to determine the value of X before implementing procedural steps for solving for X. A student might reason the 3 x's is 5 less than 38 so if I remove the 5 from both bars, I'm left with 3x at the top and 33 at the bottom. The next thought may be to consider now 33 is broken into 3 equal parts, making X equal to 11.

What equation do you think this bar diagram is representing? $4x - 3 = 25$. How might your student use this diagram to solve for x? Then how could you help them make the connection to solving it algebraically without explicitly telling them step by step?

Algebra tiles and algeblocks are great tools for visualizing equations as well. With algebra tiles, the long green tiles are X, the long red are -X, small squares are positive one or negative one or yellow and red respectively. Take a look at the model. What equation is represented? $3x - 2 = 4$. With these we could discuss what it means to isolate the variable. The negative two must become zero in order for us to remove it. This is a great place to briefly discuss zero pairs. But most importantly, do not lose the idea of balance or equivalence. Most teachers say it this way, "whatever you do to one side, you must do to the other". This is truth but the "why" has dropped off. Because the two sides are the same...then continue the discussion about what must happen on both sides.

A great place to begin the use of these tools is through a context. How could you model this problem with algebra tiles? Cuisenaire rods? Bar diagram?

Using tables to organize information from the context can help when creating a visual of the problem or moving from concrete representation to abstract.

Slide 23-44: when teaching students to solve systems of equations, we discuss three strategies for solving. Quite often, these strategies are purely procedural causing some students to have a difficult time making sense of the steps.

You may find beginning with a concrete representation will provide students a better understanding of why the strategies work.

Let's look at this example. We are representing the bowl with b and the cup with c. We have one bowl and two cups which is equivalent to 700 grams. The other quantity is two bowls and one cup equal to 800 grams. After setting up the concrete equations, students will notice each equation has at least one b and at least one c, these quantities can be removed. With probing, students will conclude in order to get to one variable to identify its value another set must be created. Here, we've duplicated bcc equivalent to 700 grams and removed the b's from each equation. What's remaining? 3 cs. At this point you would want students to conclude the value of the 3c by determining what amount was removed from what was originally created. 1400 was created and from it we have removed 800, making $3c=600$. We now know the value of c and can substitute it into one of the original equations to find the value of b.

After working with these concrete models for a while, you could make an explicit connection to the algebraic equations. We began with the given equations. We had to duplicate which equation? How many times was it duplicated? Which quantities were removed? Mathematically, to remove it, it needs to be negative. What can we do to make these quantities negative? We have eliminated a variable allowing us to solve for the other. Once we find its value, we can substitute it back into one of the original equations. Our solution for both is (200, 300).
Take a look at this problem. How could you setup a concrete model to represent this? Pause for a moment to setup your representation.
In this problem, identifying labels will be helpful. The independent variable is the one representing the muffins. Therefore, the cupcakes are represented with 2 mm's plus 4. What observations can be made? If you see 3 m's plus 4 which is equivalent to 40, you're spot on. Now we can solve for m. With the value for m we can solve for c.
Just as before, you should be explicit when making the connection to the algebraic equations. Let's see if you are able to make the connection. As we go through the written equations, discuss its connection to the concrete model.

Slide 45: what do you do when students struggle?

Slide 46: allow students to work at their own pace of understanding. It's perfectly fine for students to hover between the representational stage and abstract stage.

Slide 47: tiered tasks are a great way to allow students to work at their own pace of understanding while continuing to move through the curriculum. You may have a single task tiered at three different levels. Below expectation, at expectation and above expectation.

Slide 48: the task on literal equations from module 4 could be tiered in this manner. Notice how the equations increase in difficulty as you go through the tiers.

Slide 49: meeting with small groups of students is certainly beneficial. We have pulled suggestions for implementing small groups. The link listed will provide insight on how other teachers are implementing small groups at the secondary level.

Slide 50: this suggestion comes from the Social Emotional Learning in Mathematics session from Global Math Department. It lists a class size of 30 students to demonstrate you do not have to have a small class size to implement small group instruction. The GADOE is not suggesting the FOA class sizes should be increased to 30 students.

Slide 51: this suggestion comes from the Social Emotional Learning in Mathematics session from Global Math Department. It lists a class size of 30 students to demonstrate you do not have to have a small class size to implement small group instruction. The GADOE is not suggesting the FOA class sizes should be increased to 30 students.

Slide 52: the GA DOE has created a wiki space for high school teachers to engage in discussions about math teaching using the GSE. Here you'll find a forum to keep the conversation going, resources and who knows, you might make a few friends. It's a call to action and you are invited to make a difference. Access this online community here: https://ccgpsmathematics9-10.wikispaces.com/