Module 3: Proportional Reasoning Script

Slide 1 - This is Module 3: Proportional Reasoning, part of a professional learning series for Foundations of Algebra teachers. We encourage you to follow along and construct the visual representations discussed within this session. Feel free to pause at any time to share your thoughts, ideas, and questions with your colleagues.

Slide 2 - This professional learning series is designed to explore the Foundations of Algebra content as well as strategies for teaching that content by looking at the big ideas within this module.

Slide 3 – The first of the 4 big ideas in this module is that ratios are multiplicative comparisons. This is a key idea that students need to develop. Often, ratios are addressed in a superficial manner with students recording the symbols (7:10) to tell the ratio of cats to dogs. Ratios should be taught as relations within contexts that involve multiplicative reasoning.

Slide 4 – Sometimes, to put students at ease, teachers will tell students "Ratios are just like fractions." While ratios and fractions are similar, there are important distinctions that students should be able to make. Here are three examples to help with develop the understandings of the overlaps of these two concepts as well as the differences. Which of these relations are ratios and which are fractions? Are there any that are both? Pause the video here and discuss this with your team.
What was your conversation like? Did you construct viable arguments? Did you critique each other's reasoning? Let's see how you did:

> The first example is a ratio, not a fraction because fractions are not part-to-part ratios.
> The second example can also be thought of, as three-tenths of the pets are dogs. Since this is a part-to whole ratio, it is both a ratio and a fraction.
> The last example is a fraction of a length and is not a ratio because there is not a multiplicative comparison.

Slide 5 – Ratios can actually be thought of in 4 ways:
Part to Part: In this type of ratio, one part of a whole (8 red counters) can relate to another part of the same whole (5 blue counters). This can be represented as $\frac{8}{5}$, or a ratio of 8 to 5 (not eight-fifths) because it is not a fraction even though it can be written with the fraction bar. We see part to part ratios in the diagonal of a square to its side as $\sqrt{2}$, the slope of a line as a ratio of rise for each unit of horizontal distance and when finding the odds of an event occurring.
Part to Whole: Ratios can be written as a part of a whole (8 red counters) to the whole group of counters (13). This can be thought of as $\frac{8}{13}$ of the counters are red (a fraction). Percentages and probabilities are part to whole ratios.
Ratios as Quotients: If you can buy 10 candies for \$1.00, the ratio of money for candies is \$1.00 to 10 candies. The cost per candy piece is \$0.10 (the unit rate).
Ratios as Rates: miles per gallon, flowers per bouquet, and cans per case are all rates.

Rates involve 2 different units and how they relate to each other. Feet per yard, centimeters per inch, and milliliters per liter are also rates. A rate represents an infinite set of equivalent ratios.

Slide 6 – There are two ways to think about Ratio.
A multiplicative comparison. Consider the two pencils on the left. The lengths of the pencils are 6 inches and 8 inches. At this point, it's common to ask for a ratio of the pencil lengths. When students tell us 6 to 8. We assume they know ratios and move on. This is a very low level question and does not communicate the relationship between the measures of the pencils. Pause the video here and discuss with your team how these pencils could be compared multiplicatively.

Did your conversation include multiplicative relationships like these:

The short pencil is six-eighths as long as the long pencil (or three-fourths as long).
The long pencil is eight-sixths as long as the short pencil (or four-thirds or $1\frac{1}{3}$ the length).

The multiplicative relationship is the ratio (not the symbols) and involves us asking questions like:
"What fractional part is one thing of another?" or " How many times greater is one thing than another?

A composed unit. This refers to thinking of a ratio as one unit. This is a key development milestone for students. Unitizing is not a new idea for students, but every time it shows up, students need to grapple with it before owning the idea. If packs of crayons are 4 for $1.00, you can think of this as a unit as well as multiples of this unit like 8 for $2.00, 20 for $5.00, etc. We can also partition the composed unit to 2 for $0.50 and 1 for $0.25. Any number of packs of crayons can be priced using composed units.

Slide 7 – It's important for students to be able to apply both types of ratios. In the cupcake problem, the context allows for students to first chew on the idea of composed units, and then multiplicative comparison. Pause the video here and try this problem yourself. Give yourselves some alone time and some together time with this problem.

Did you all attack this problem in the same way? Did you use materials like colored tiles at any point, or drawings? If no one used materials or drawings, pause the video now and take a few minutes to see if you can make sense of this problem in this way. Given options to use tools or drawings, students will make sense of problems like this and develop these two ways to think about ratios.

Slide 8 – The second big idea is about equivalent ratios and how the comparisons involve multiplication and division rather than addition and subtraction.

Slide 9 – Problems like these two can be problematic for students. Sometimes classes as a whole will say they're both the same. Each set has two girls. It's important, when helping students to develop multiplicative thinking, that you run through these problems as a student. Jot down all possible arguments students might give. Then think of some least helpful questions (not leading questions) to ask and write those down as well. This is much easier when done before the lesson. You may not think of all they might say, but it sure helps to have some questions written down. Pause here and discuss with your team what arguments students might make (even if the only one you expect is that they'll say, "They both have 2 girls." or "They both have 3 circles.") Then take some time to write some least helpful (not leading) questions to promote student thinking.

Welcome back. Did you come up with more than one argument students might make? What were some of the questions you asked? Were they least helpful?

Here are a few questions we thought up:
What are some other ways to compare sets?
Ms. _____ down the hall showed this to her class and one of the students said that one group definitely had more girls. What do you think about that? How could this student be thinking about these sets differently?
Another way to start this activity might be to not ask which has more, but just leave it open and ask "What do you notice?" then write down everything they say. My guess is that someone will say the bottom group is half boys and half girls. That's the first step!

Slide 10 – For this next activity, we move students more to numeric strategies. Look at the cards. The task is to match cards where the ratio of objects is the same. It makes sense to think of these as boxes per truck rather than trucks per box. Pause the video and take a few minutes to try this, then share your strategies.

Slide 11 – This is probably my favorite task for thinking with ratios and proportions. Please pause this video and engage in this task, then share your strategies. How many ways did your team find to determine which pitcher was lemonier? Did anyone use part to whole ratios or part-to-part ratios?

Slide 12 – This task comes from Illustrative Mathematics. The great thing about this task is that it comes from a real ticket booth (you can see the picture at https://www.illustrativemathematics.org/content-standards/6/RP/A/2/tasks/1975) and it also leads us to the next big idea. Take a few minutes to tackle this task and share your results with your team.

Slide 13 – The third big idea in this module is the idea of rates and how they represent an infinite number of equivalent ratios.

Slide 14 – A nice way to introduce this idea, after a lot of practice with reasoning about ratios like what we've seen so far, is with a graphing story. A graphing story asks students to graph what they see happening in a 15 second video. Students will need a sheet like this and a video (all of these can be found at http://graphingstories.com/). If you have some of these sheets handy, grab some, if not, grab some graph paper and make it look like one of these graphs. Time is along the x-axis (seconds). The y-axis varies from story to story, but pay attention to the beginning of each video and you can quickly jot down the meaning of and values on the y-axis. You can also pause the video for students.

Slide 15 & 16 – Here we go. (When the video ends): Pause here to take some time to share your graphs and talk about any differences/similarities as well as what students may do.

Slide 17 – As you probably know, explorations of rate help students build the concept of slope, which describes the rate of change for a linear function. Conceptually, we're developing the idea that slope is the rate that shows how much y increases when x increases by 1.

Try this:

Take a look at these two lines. What do you notice? What do you wonder? Pause here and jot down all of your notices and wonders and then share with your team.
Are the lines parallel? What information would you need to help settle this?

(If only there was some way to count and see how much one line was rising.) Maybe a tool, like a grid might help? Here you go. Anything else you might want? Axes? How about some points? Is this helpful?

Note: When doing something like this with students, it's best to give only the information that they ask for in order to foster a productive struggle.

Are the lines parallel? If not, which line is climbing at a faster rate and will they intersect above or below our field of view? If so, how do you know they will never intersect?

Slide 18 & 19 – What about no slope and zero slope?
These are two easily confused slopes and like many ideas in mathematics, contexts can help students distinguish between the two.

Consider this story:
You walk for 5 minutes at a rate of 1 mph. You stop for 3 min to watch some ducks in the pond. Then you walk for 8 more minutes at 2 mph.

What will the graph look like for the three minutes you stopped? What is your rate when you stop?

What if you saw this graph of a walking story? What would the vertical line mean? You traveled a distance without time passing! Remember, rate is based on a change of 1 in the x variable.

Slide 20 – The fourth and final big idea for module 3 is all about solving proportions in a variety of contexts through the use of reasoning rather than fixed procedures.

Slide 21 – Back in module 2, we looked at some growing patterns. Here are three more. Do these growing patterns describe proportional relationships? Take a few minutes to discuss these patterns with your team. Graph them. What do you notice? Which are proportional and which are not.

Revisiting concepts that relate in familiar contexts like this helps students understand the interrelatedness by giving students the opportunity to make connections.

Slide 22 – Textbooks typically show students how to set up an equation of two ratios involving an unknown. The challenge we face is to teach ideas and reasoning rather than a "quick and easy" computation algorithm.

Rather than tell students to cross multiply, build understanding through visuals like a number line diagram seen here. This visual allows for multiple strategies to emerge, based on student reasoning. For example, students may find the price of 1 pound of apples by dividing, and then multiply that by 6. These steps can be shown on the number line itself. Another student may determine that the scale factor here is 1.5 and multiply this by 4.

Slides 23 – 25 – Try to solve this 3-Act task by Andrew Stadel with your team using a strategy other than cross products. A link to all of Andrew's tasks can be found here: http://mr-stadel.blogspot.com/p/3-act-catalog_17.html. A copy of a 3-Act recording sheet adapted by Graham Fletcher can be found here: https://gfletchy.files.wordpress.com/2013/09/3-act-recording-sheet1.pdf

You know the drill… Take a few minutes to jot down your notices and wonders and share with your colleagues.

The anticipated question here is: "How long is that song?"

Make your "best guess" estimate. In the next 5 seconds, write your "best guess" estimate. 5, 4, 3, 2, 1.

Now, write an estimate that is too low, and one that is too high.

Write your too high and too low estimates at the ends of a number line. Now, place your best guess estimate where you think it belongs. This is a nice benchmark to assess your students' understanding of the magnitude of numbers.

Act 2: What information do you need to answer this question?

Since I can't wait for you to ask, let me show you an image.

Try to use this information to answer the question, "How long is that song?"

Pause here to give you and your team some time to solve the problem.

How did you do? What strategies did you use? Watch the reveal.

Why might your answer be off?

Slide 26 – To put all of this together requires three ideas.

Slide 27 – The first is differentiation. Think CRA – Concrete – Representational – Abstract. And that doesn't mean for the 1st three days we use manipulatives, then put them away and just draw pictures for three days, then finally, we use an algorithm. All three must be included from the start. Just like with the growth patterns. The sketch is included the first time they investigate, but there's also an expression at the end. It may not be algebraic the first time, but students will work up to that understanding.

Slide 28 – Here are some tiered tasks for working with ratios and proportions. Students can all work on the same standard with various levels of support. Students at all levels benefit from rich problems like How many fish in the pond? to start. To provide support for those below level students, supplement with tasks such as mixing colors from http://nzmaths.co.nz/resource/mixing-colours

Slide 29 – Meeting with small groups of students is certainly beneficial. It can be overwhelming to schedule. The two slides that follow offer some suggestions about how to manage small groups so as to maximize student learning.

Slide 30 – This suggestion comes from the Social Emotional Learning in Mathematics session from Global Math Department. It lists a class size of 30 students to demonstrate you do not have to have a small class size to implement small group instruction. The GADOE is not suggesting the FOA class sizes should be increased to 30 students.

Slide 31 – This suggestion comes from the Social Emotional Learning in Mathematics session from Global Math Department. It lists a class size of 30 students to demonstrate you do not have to have a small class size to implement small group instruction. The GADOE is not suggesting the FOA of algebra class sizes should be increased to 30 students.

Slide 32 – Let's keep the conversation going! The DOE has created a wikispace for high school teachers to engage in discussions about math teaching using the GSE. Here you'll find a forum to keep the conversation going, resources and who knows, you might make a few friends. It's a call to action and you are invited to make a difference! Access this online community here: https://ccgpsmathematics9-10.wikispaces.com/

Slide 33 – Speaking of online communities… If you're on Twitter, join the #MTBoS (Math Twitter Blog-o-Sphere) where everyone has a voice and we all learn from each other. There are no dues, no applications. If you're a math teacher and you're on twitter, you're in! If you're not on twitter, then this is your reason to join. If you're not a math teacher, become one! https://exploremtbos.wordpress.com/
https://twitter.com/