Module 2: Arithmetic to Algebra Script

Slide 1 - This is module 2: Arithmetic to Algebra, part of a professional learning series for Foundations of Algebra teachers. We encourage you to follow along and construct the visual representations discussed within this session. Feel free to pause at any time to share your thoughts, ideas, and questions with your colleagues.

Slide 2 - This professional learning series is designed to explore the Foundations of Algebra content and strategies for teaching that content by looking at the **big ideas** within this module.

Slide 3 – The first of the 4 big ideas in this module is that Algebra is useful in that it helps us make sense of the patterns in our world. This big idea involves two things that all math teachers should strive to engage their students in every day.

Slide 4 – The first of these are the Standards for Mathematical Practice. These 8 practices are a guide as to HOW we should be teaching the content of any math course. Students engaging in these practices on a daily basis must see mathematics as useful, not because we say it is, but because they are using it to make sense.

Slides 5-10 - The second is Context. Contextual problems provide students at all levels a place to start. Contexts in mathematics provide the opportunity for students think and practice becoming better thinkers. Contexts may involve words, or just pictures or videos. Look at the patterns in Sierpinski's Triangle or the conch shell. Contexts for mathematical learning are all around us every day. We just have to present them to our students. What do you wonder when you see this picture? Three act tasks like this one can also provide contexts for learning. Contextual problems and engaging students in the eight mathematical practices gets students interested in mathematics which leads to them appreciating it as useful and worthy.

Slide 11 – The next big idea has to do with the symbols of mathematics. Two of the most misunderstood are the equal sign (as well as the < and > symbols) and variables. In order be successful, students need to be given multiple opportunities to build understandings of these two ideas conceptually.

Slide 12 - To build conceptual understanding of the equal sign while building other algebraic understandings, provide a context such as a balance. Press pause and take a minute to jot down what you notice and wonder about this scale. Share with your colleagues and press play when ready. Much of what you notice and wonder, will be noticed and wondered by your students as well. Wonders from students will likely be: Which one weighs the most? The least? Once you have one or both of those questions, ask them to work together to figure it out. The discussions that come from these simple contexts will likely be some of the richest!

Slide 13 – 15 – Follow up with other equivalence ideas such as these and others on line at https://solveme.edc.org/Mobiles.html. Ideally, it would be best to use real scales, however the idea of how scales work provide students that starting point to strategize and problem solve successfully.

Slide 16 – (video clip) - Another idea for engaging students in conceptually understanding variables and constants is through number tricks. Try this with me now:

- Think of a number.
- Add 3.
- Double your total.
- Subtract 4.
- Divide the result by 2.
- Subtract the number you first thought of.
- You got the number 1!

Now, take a look at one model. See if the representations help you understand this number trick on another level.

Finally, take a look at what picture representations can do to develop students' understanding of not only variables but also what it means to undo the steps to one of these number tricks.

These picture representations help build early understandings of solving equations on a conceptual level. And to think… it all started with a little number trick!

Slide 17-20– Number tricks like these can be found in Module 2. One is A Math-ic Prediction which builds nicely off of a number trick like the one we just saw. What's nice about this task is that it uses one of my favorite manipulatives – sticky notes! Using sticky notes instead of bags is nice because they're cheap, easier to sketch, and any number can be easily written on a sticky note.

Consecutive number sums and Triangle Mystery were both created using scratch, a free online coding platform. Students can take a look inside of each of these and manipulate the code to create their own number tricks. Then they can build one of their own…from scratch! https://scratch.mit.edu/

Slide 21 – The third big idea deals generalizing our computational structures. Most of the content in this module falls within this big idea.

Slide 22 – True/False Sentences. Take a look at these sentences. Which are true and which are false? Press pause and engage in this activity. Be sure to provide reasons for your choices.
As mentioned earlier, the meaning of the equal sign needs strong understanding from the conceptual level. True/False explorations help clarify this meaning and that's just one of the outcomes.

It's important to engage students in the 8 mathematical practices, so listen to students' reasoning for their choices. Also, for false statements, ask students to rewrite the statement with > or < to make the statement true. This provides students more opportunities to reason about quantity, make sense of the equal sign. Allowing students to share their thoughts, gets them out of their own heads and gets them talking like mathematicians. With enough practice, they begin thinking like mathematicians.

Slide 23 – Open Sentences – After students have some experience with True/False Sentences, introduce them to open sentences such as those shown here. Again, press pause and share your reasoning about how you figured out the number for each box. You may be surprised at what you hear from your colleagues.
After using open boxes, replace them with a variable.

Slide 24 – Relational thinking happens when students observe and use numeric relationships between two sides of the equal sign rather than actually computing the amounts on each side.

Slide 25 – For example take a look at the first True/False sentence. Thinking relationally, I can tell that this is true because the start numbers 6.74 and 6.64 are 0.1 apart and the end numbers 3.89 and 3.79 are also 0.1 apart. So the distance between the points on the number line didn't change.

Slide 26 – Likewise with these open sentences. Take a moment to discuss these with your colleagues. How would you find the value of the box without computing? How well do you think relationally? Tasks like these empower students. Once introduced, these can be given to students on a consistent basis as part of a classroom routine.

Slide 27 – Exponents used to be one of my least favorite topics to teach, because they led to scientific notation! Recently, I had the chance to hear how another teacher dealt with this same issue. Instead of giving students exponent rules for memorization, he thought about all of the mistakes students make, created a worksheet with all of these mistakes and told them all up front that all of the statements were incorrect. The students' task was to identify the mistake, correct it, then justify their reasoning. Students had 3-5 minutes of individual work time and then shared ideas with their group. This was all followed by a classroom discussion. Andrew Stadel wrote about this on his blog. In this post, he explains in more detail what happened and provides 2 more days worth of this type of exploration. His post can be found here: http://mr-stadel.blogspot.com/2013/04/thank-you-math-mistakes.html

Slide 28 – Square and cube roots can seem daunting and even tedious – especially if students are asked to memorize a list of them! They don't have to be. Students can be engaged in the 8 math practices while engaging in a contextual problem such as this. To begin, ask as before: What do you notice? Then write down everything they say. Pause here and take a minute now to do this with your colleagues. Jot down your notices. What did you see? Did you notice the multiplication table? Did you notice that the colors form a backward "L?" The squares are on the diagonal? These are all things students notice and more! But where is the square root? Do you notice that the square is near the center and all of the other pieces of the same color are just as long as the side of the square? That's the root. The side of the square is the root of the square or the square root.

So, what about cube roots? If you ask students what they wonder about this picture (and you should, really) someone will say something like, "Why are they colored that way?" If you don't have some cubes handy, get some and look at the color groups. What do you notice? You're not going to believe it when you see it!

Slide 29 - The Pythagorean relationship states that if a square is constructed on each side of a right triangle, the areas of the two smaller triangles will together equal the area of the square on the hypotenuse. This is one of the most important mathematical relationships and demands in-depth conceptual investigation.

One possible way to begin this conceptual investigation might be to assign pairs of students a different right triangle by specifying the lengths of the two legs.

(Video Clip) Have students draw their right triangle on grid paper. Next, students are to draw squares on each leg and on the hypotenuse.

To construct the square on the hypotenuse, students can think of the hypotenuse as the diagonal of a rectangle. Then, they can draw 3 more congruent rectangles that form a large square. By connecting the diagonals of these rectangles, they create a square whose sides are congruent to the hypotenuse.

Students collect data from their classmates and create a table of data: area of square on leg a, area of square on leg b, and area of the square on the hypotenuse. The task is to look for a relationship between the squares.

Slide 30 – The last of the big ideas for this module involves recognizing, extending, and generalizing patterns.

Slide 31 – Take a look at this growing pattern. What do you notice? Though this may be a simple growth pattern, it may be useful to dig a little deeper into how it grows, allowing students to experience multiple ideas.

Slide 32 – For example, when asked to change the color of the sticky notes that seem to stay the same in this growth pattern, students may share one of these ideas. Build a table with students, asking them, what data we can collect. You'll get something like this on the right. Note that the columns are labeled "stays the same" and "changes." These are names that students usually come up with due to the question asked earlier. If one color represents those that stay the same, the others must be changes. Once a table like this is filled to stage 5 or 6, ask students what they notice. After they share all of the patterns they see, skip a few stages to facilitate the development of an explicit rule for this pattern. Then skip a few more. Finally, place a sticky note under the stage column. Ask students if they could figure out how many sticky notes it would take for any stage number you write on the post it? Work your way through the table for the sticky note. The changes column for this example would be sticky note + 1. Finally, remember that picture representations, though not included in this example from the module 2 frameworks often help deepen understandings. They're also a record of thinking that students can easily return to…

Slide 33 – As you can see in this student work sample.

Slide 34 – Many other free patterns to use can be found here http://www.visualpatterns.org

Slide 35 – To put all of this together requires three ideas.

Slide 36 – The first is differentiation. Think CRA – Concrete – Representational – Abstract. And that doesn't mean for the 1st three days we use manipulatives, then put them away and just draw pictures for three days, then finally, we use an algorithm. All three must be included from the start. Just like with the growth patterns. The sketch is included the first time they investigate, but there's also an expression at the end. It may not be algebraic the first time, but students will work up to that understanding.

Slide 37 – Here are some tiered tasks for working with growth patterns. Students can all work on the same standard and write equivalent algebraic expressions.

Slide 38 – Meeting with small groups of students is certainly beneficial. It can be overwhelming to schedule. The two slides that follow offer some suggestions about how to manage small groups so as to maximize student learning.

Slide 39 - This suggestion comes from the Social Emotional Learning in Mathematics session from Global Math Department. It lists a class size of 30 students to demonstrate you do not have to have a small class size to implement small group instruction. The GADOE is not suggesting the FOA class sizes should be increased to 30 students.

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Slide 41 - Let's keep the conversation going! The DOE has created a wikispace for high school teachers to engage in discussions about math teaching using the GSE. Here you'll find a forum to keep the conversation going, resources and who knows, you might make a few friends. It's a call to action and you are invited to make a difference! Access this online community here: https://ccgpsmathematics9-10.wikispaces.com/

Slide 42 – Speaking of online communities… If you're on Twitter, join the #MTBoS (Math Twitter Blog-o-Sphere) where everyone has a voice and we all learn from each other. There are no dues, no applications. If you're a math teacher and you're on twitter, you're in! If you're not on twitter, then this is your reason to join. If you're not a math teacher, become one! https://exploremtbos.wordpress.com/ https://twitter.com/