Module 2 - Arithmetic to Algebra
Big Ideas
1. *Algebra* is a useful tool for generalizing arithmetic and representing patterns in our world.
Habits of a Productive Mathematical Thinker

**Standard of Mathematical Practice #1**
Make sense of problems and persevere in solving them

**Standard of Mathematical Practice #6**
Attend to Precision

Reasoning and Explaining

**Standard of Mathematical Practice #2**
Reason abstractly and quantitatively

**Standard of Mathematical Practice #3**
Construct viable arguments and critique the reasoning of others

Modeling Using Tools

**Standard of Mathematical Practice #4**
Model with mathematics

**Standard of Mathematical Practice #5**
Use appropriate tools strategically

Seeing Structure and Generalizing

**Standard of Mathematical Practice #7**
Look for and make sense of structure

**Standard of Mathematical Practice #8**
Look for and express regularity in repeated reasoning
CONTEXT
Source: http://robertkaplinsky.com/
CONTEXT
2. **Symbols**, especially those involving equality and variables, must be well understood conceptually for students to be successful in mathematics, particularly algebra.
What do you notice? What do you wonder?
What will balance 2 spheres?
How much does each shape weigh?
## Number Tricks

<table>
<thead>
<tr>
<th>Words</th>
<th>Pictures</th>
<th>Lucy</th>
<th>Alex</th>
<th>Mila</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think of a number.</td>
<td><img src="image" alt="Bag" /></td>
<td>5</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>Double it.</td>
<td><img src="image" alt="Bag" /> <img src="image" alt="Bag" /></td>
<td>10</td>
<td>18</td>
<td>28</td>
</tr>
<tr>
<td>Add 4.</td>
<td><img src="image" alt="Bag" /> <img src="image" alt="Bag" /> <img src="image" alt="Blue Circles" /></td>
<td>14</td>
<td>22</td>
<td>32</td>
</tr>
<tr>
<td>Divide by 2. What did you get?</td>
<td><img src="image" alt="Bag" /> <img src="image" alt="Blue Circles" /></td>
<td>7</td>
<td>11</td>
<td>16</td>
</tr>
</tbody>
</table>

It's hard to undo... using **JUST** the words!
<table>
<thead>
<tr>
<th>Words</th>
<th>Pictures</th>
<th>Diagrams</th>
<th>Hailee</th>
<th>Connor</th>
<th>Lura</th>
<th>Maury</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think of a number</td>
<td>![Yellow Image]</td>
<td></td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Double it</td>
<td></td>
<td></td>
<td></td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Add 8</td>
<td></td>
<td></td>
<td></td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Divide by 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
</tr>
</tbody>
</table>
Consecutive Number Sums

Start Number

Algebra

Magic Trick

+ 0

0

0

0

0
Triangle Mystery

Start Number
https://scratch.mit.edu/
3. The methods we use to compute and the structures in our number system can and should be generalized. For example, the generalization that \( a + b = b + a \) tells us that \( 83 + 27 = 27 + 83 \) without computing the sums on each side of the equal sign.
True or False Sentences

$120 = 60 \cdot 2$

$1 = \frac{3}{4} + \frac{2}{1}$

$318 = 318$

$\frac{1}{2} = \frac{1}{4} + \frac{1}{4}$

$347 + 71 = 70 + 348$

$1210 - 35 = 1310 - 45$

$0.4 \cdot 15 = 0.2 \cdot 30$
Open Sentences

0.5 + □ = 5
0.4 + □ = 0.6
4.5 + 5.5 = □ − 1

0.3 · 7 = 7 · □
□ · 4 = 4.8
□ = 2.3 − 0.5
Relational Thinking

True/False
Relational Thinking

True/False

\[
6.74 - 3.89 = 6.64 - 3.79 \quad \quad \quad 42 = 0.5 \cdot 84
\]

\[
\frac{2}{5} = \frac{1}{3} + \frac{1}{2}
\]

\[
64 \div 14 = 32 \div 28
\]

Open Sentences

\[
7.03 + 0.056 = 7.01 + \square \quad \quad \quad 0.126 - 0.37 > \square - \square
\]
\[
\frac{2}{5} = \frac{1}{3} + \frac{1}{2} \\
64 \div 14 = 32 \div 28
\]

Open Sentences

\[7.03 + 0.056 = 7.01 + \square\]

\[0.126 - 0.37 > \square - 0.4\]

\[20 \cdot 4.8 = n \cdot 2.4\]

\[\frac{2}{5} + \frac{1}{5} = \frac{3}{10} + \frac{1}{10} + n\]
The following statements are all INCORRECT.
1. Identify the mistake(s).
2. Correct.
3. Justify (show) your reasoning.

1. \(2^5 = 10\)  
5. \(100^{1/2} = 50\)

2. \((-2)^3 = 8\)  
6. \(7^{-2} = -49\)

3. \(-6^2 = 36\)  
7. \((y^3)^5 = y^8\)

4. \(37^0 = 0\)  
8. \(\frac{x^5}{x^9} = x^4\)

Sources: Andrew Stadel http://mr-stadel.blogspot.com/  
Michael Pershan http://mathmistakes.org/
4. *Patterns*, both repeating and growing, can be recognized, extended, and generalized.
Growing Patterns
This column allows students to keep a record of their thinking. If they run out of time, they can pick up where they left off later without having to start from scratch.

<table>
<thead>
<tr>
<th>Step (Stage)</th>
<th>Sketch</th>
<th>Stays Same</th>
<th>Changes</th>
<th>Total (tiles in this case)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>1</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>1</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>1</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>37</td>
<td></td>
<td>1</td>
<td>38</td>
<td>39</td>
</tr>
<tr>
<td>n</td>
<td></td>
<td>1</td>
<td>n+1</td>
<td>n + 1 + 1</td>
</tr>
</tbody>
</table>

http://www.visualpatterns.org/
<table>
<thead>
<tr>
<th>Stage #</th>
<th>Constant</th>
<th>Changes</th>
<th>Sketch</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td><img src="image" alt="Stage 1 Sketch" /></td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>6</td>
<td><img src="image" alt="Stage 2 Sketch" /></td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>9</td>
<td><img src="image" alt="Stage 3 Sketch" /></td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>12</td>
<td><img src="image" alt="Stage 4 Sketch" /></td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>15</td>
<td><img src="image" alt="Stage 5 Sketch" /></td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>30</td>
<td></td>
<td>31</td>
</tr>
<tr>
<td>50</td>
<td>1</td>
<td>150</td>
<td></td>
<td>151</td>
</tr>
<tr>
<td>n</td>
<td></td>
<td>n×3+1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Meeting Students' Needs

Differentiation:
C-R-A at their own pace
Allow students their own time to build their understanding. They will be in different places of understanding which is normal. It is okay for students to remain in the representational stage for an extended period until they are ready to move to abstract thinking.

Tiered Tasks

Small Group Instruction
Differentiation:
C-R-A at their own pace

Allow students their own time to build their understanding. They will be in different places of understanding which is normal. It is okay for students to remain in the representational stage for an extended period until they are ready to move to abstract thinking.
Tiered Tasks

Above

1  2  3

At

1  2  3  4

Below

Small Groups
Small Group Instruction

Option 1 - 45 minutes
30 students, 4 groups
- Monday - whole group lesson, mini-lesson, student-centered work session
- Tuesday - small group rotations
- Wednesday - small group rotations
- Thursday - formative assessment, whole group lesson

Option 2 - 60 minutes
30+ students, 4 groups
- Monday - whole group, mini-lesson, student-centered, independent practice
- Tuesday - small group rotation
Option 1 - 45 minutes
30 students, 4 groups

- Monday - whole group lesson, mini-lesson, student-centered work session
- Tuesday - small group rotations
- Wednesday - small group rotations
- Thursday - Formative assessment, whole group lesson
- Friday - Common formative assessment

NOT typical class size for FOA. Remember this suggestion comes from the GMD session.
Option 2- 60 minutes

30+ students, 4 groups

- Monday- whole group, mini-lesson, student-centered, independent practice
- Tuesday- small group rotation
- Wednesday- small group rotation
- Thursday- Formative assessment, whole group lesson
- Friday- Common formative assessment

NOT typical class size for FOA. Remember this suggestion comes from the GMD session.
https://ccgpsmathematics9-10.wikispaces.com/
Module 2 - Arithmetic to Algebra