Module 1: Use of Manipulatives and Strategies Part 1

Focus on Fractions
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Set Models

What defines the whole:
Whatever value is determined as one set

What defines the parts:
Equal number of objects

What the fraction means:
The count of objects in the subset, as it relates to the defined whole.
Let's look at some examples of set models.
*Think about: If 12 counters are three-fourths of a set, how many counters are in the full set?
Length Models

What defines the whole:
The unit of distance or length

What defines the parts:
equal distance/length

What the fraction means:
The location of a point in relation to 0 and other values on the number line.
Examples of linear models:
- cuisenaire rods
- strips of paper
- adding machine tape
- number lines
Multiplying fractions using a number line:
\frac{1}{2} of \frac{1}{4}
\frac{1}{4} \text{ of } \frac{2}{3} = \frac{2}{3}
\( \frac{1}{4} \text{ of } \frac{2}{3} \)
\[ \frac{1}{4} \text{ of } \frac{2}{3} = \frac{1}{6} \]
\[ \frac{2}{5} \text{ of } \frac{1}{3} \]
$\frac{2}{5}$ of $\frac{1}{3}$
\[
\frac{2}{5} \text{ of } \frac{1}{3} = \frac{2}{15}
\]
Dividing fractions with number lines:
\[\frac{4}{5} \div \frac{1}{5}\]
\[ \frac{4}{5} \div \frac{1}{5} = 4 \]
$1\frac{1}{3} \div \frac{2}{3}$
\frac{3}{8} \div \frac{1}{4} = 1 \frac{1}{2}
Dividing fractions using cuisenaire rods:
\[ \frac{1}{2} \div \frac{1}{4} = 2 \]
*Think about:
How could you model 1 1/3 divided by 2/3?

Using a length model, model 1 1/2 of 2 3/4.
Area Models

What defines the whole:
The area of the defined region

What defines the parts:
Equal area

What the fraction means:
The part of the area covered, as it relates to the whole unit
Folding paper:

One whole paper

Folded into 4 equal parts
One whole paper
Folded into 4 equal parts
*Think about: How would you use an area model to solve three-fifth of three-fourths?
Abstract Thinking

With enough experience using area model or linear models, students will start to notice a pattern.
Look at the three problems below:

\[
\begin{align*}
5/6 \times 1/2 \\
3/4 \times 1/5 \\
1/3 \times 9/10
\end{align*}
\]

With these problems, students can identify a pattern by first modeling the expressions using an area model. Then ask, "How did you figure out that the denominator would be twelfths? Is this a pattern that is true for the other examples?"
Let's look at multiplying fractions greater than 1 (partial products):

A mixed number can be decomposed into a sum of unit fractions. (The definition of a mixed number is a whole number plus a fraction.)
A mixed number can be decomposed into its two addends. (The definition of a mixed number is a whole number plus a fraction.)
$3 \frac{1}{3} \times 2 \frac{1}{4}$

wholes $3 \times 2$

fraction/whole $3 \times \frac{1}{4}$

$2 \times \frac{1}{3}$

Fractions $\frac{1}{3} \times \frac{1}{4}$
$3\frac{1}{3} \times 2\frac{1}{4}$

wholes $3 \times 2 = 6$

fraction $3 \times \frac{1}{4} = \frac{3}{4}$

whole $2 \times \frac{1}{3} = \frac{2}{3}$

Fractions $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$
\[ \frac{3\frac{1}{3}}{2\frac{1}{4}} \times 6 + \frac{3}{4} + \frac{2}{3} + \frac{1}{12} \]

\[ 6 + \frac{9}{12} + \frac{8}{12} + \frac{1}{12} \]

\[ 6 + \frac{18}{12} \]
\[ 6 + \frac{18}{12} \]
\[ 6 + \frac{12}{12} + \frac{6}{12} \]
\[ 6 + 1 + \frac{6}{12} \]
\[ 7 + \frac{6}{12} \]
\[ 7\frac{6}{12} \]
Common-Denominator Algorithm for division:

\[
\frac{7}{8} \div \frac{3}{8} = 2\frac{1}{2} \\
\frac{7}{8} \div \frac{3}{8} = 2\frac{1}{2} \\
\frac{4}{9} \div \frac{2}{3} = \frac{8}{9} \\
\frac{4}{9} \div \frac{2}{3} = \frac{8}{9} \\
\frac{4}{9} \div \frac{2}{3} = \frac{8}{9} \\
\frac{4}{9} \div \frac{2}{3} = \frac{8}{9}
\]
\[
\frac{7}{8} \div \frac{3}{8} = 2 \frac{1}{3}
\]
\( \frac{7}{8} \div \frac{3}{8} = \frac{2}{3} \)

\( 3 \sqrt[7]{\frac{2}{6}} = 2^{\frac{1}{3}} \)
\[
\frac{4}{9} \div \frac{2}{3}
\]
\[
\frac{4}{9} \div \frac{2}{3} = \frac{4}{9} \div \frac{6}{9} = \frac{4}{6} \text{ or } \frac{2}{3}
\]
\[ \frac{4}{9} \div \frac{2}{3} = \frac{6}{9} \]
\[
\frac{4}{9} \div \frac{2}{3}
\]

\[
\frac{4}{9} \div \frac{6}{9}
\]

\[
\frac{4}{6} \text{ or } \frac{2}{3}
\]
Invert and Multiply Algorithm for division:

\[
1 \div 1/2 = 2 \\
3 \div 1/2 = 6 \\
4 \div 1/2 = 8 \\
5 \div 1/2 = 10
\]

Patterns you should have noticed:
There are 2 halves in 1 whole.

If there are 2 halves in 1 whole, 3 wholes has 6 halves. Multiplying the dividend by the divisor's denominator will result in the same answer.

If there are 2 halves in 1 whole, 4 wholes has 8 halves. Multiplying the dividend by the divisor's denominator will result in the same answer.
$1 \div \frac{1}{2}$

$3 \div \frac{1}{2}$

$4 \div \frac{1}{2}$

$5 \div \frac{1}{2}$
Patterns you should have noticed:
There are 2 halves in 1 whole.

If there are 2 halves in 1 whole, 3 wholes has 6 halves. Multiplying the dividend by the divisor's denominator will result in the same answer.

If there are 2 halves in 1 whole, 4 wholes has 8 halves. Multiplying the dividend by the divisor's denominator will result in the same answer.

If there are 2 halves in 1 whole, 5 wholes has 10 halves. Multiplying the dividend by the divisor's denominator will result in the same answer.
\[ \frac{2}{2/3} \]
$2 \div \frac{2}{3}$
$2 \times 3 = 6$

$6 \div 2 = 3$

$2 \times \frac{3}{2}$
Meeting Students' Needs

Differentiation

Tiered Tasks

Small Group Instruction
https://jenisesexton.wordpress.com/?s=small+group
C-R-A at their own pace
concrete-representational-abstract

Allow students their own time to build their understanding. They will be in different places of understanding which is normal. It is okay for students to remain in the representational stage for an extended period until they are ready to move to abstract thinking.
Meeting Students' Needs

Differentiation

Small Group Instruction
https://jenisesexton.wordpress.com/?s=small+group

Tiered Tasks

Below, Middle, Above

When 62 athletes in a college basketball team were asked, "How many points will our team score?"

Middle 62 athletes were asked, "How many points will our team score?"

Above 62 athletes were asked, "How many points will our team score?"

How many points will our team score according to the above graph?
Below, At and Above

A Chance Surgery

Ellifton is a surgeon at Children’s Hospital in Charleston. In 2012, he and his team performed a first-of-its-kind heart transplant for a young boy with hypoplastic left heart syndrome.
Below, At and Above

*Chance Surgery*

**Tier 1:** Dr. Clifton is a surgeon at Children’s Healthcare of Atlanta Egleston. In 2012, he and his team performed 150 surgeries to treat biliary atresia. Studies have shown that \(\frac{2}{3}\) of the patients treated for biliary atresia eventually need a liver transplant. Of Dr. Clifton’s patients last year how many will eventually need a liver transplant? According to the statistic, how many patients will not need a transplant?

**Tier 2:** Dr. Clifton is a surgeon at Children’s Healthcare of Atlanta Egleston. In 2012, he and his team performed 15 surgeries to treat biliary atresia. Studies have shown that \(\frac{2}{3}\) of the patients treated for biliary atresia eventually need a liver transplant. Of Dr. Clifton’s patients last year how many will eventually need a liver transplant? According to the statistic, how many patients will not need a transplant? Show your understanding two different ways.

**Tier 3:** Dr. Clifton is a surgeon at Children’s Healthcare of Atlanta Egleston. In 2012, he and his team performed 6 surgeries to treat biliary atresia. Studies have shown that \(\frac{2}{3}\) of the patients treated for biliary atresia eventually need a liver transplant. Of Dr. Clifton’s patients last year how many will eventually need a liver transplant? According to the statistic, how many patients will not need a transplant? Use an area model or length model to support your answer.
Meeting Students' Needs

Differentiation

Small Group Instruction
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Tiered Tasks

Table, Tier, and Active

Team A: You have 2 dollars. You want 3 dollars. How many apples will you need to trade?

Team B: The table is aligned in California's Math Standards. It uses a $/x=1$ approach to solve the problem. If you need 6 apples, how many points will you need to trade?

Team C: You have a group of students who are struggling with math. They need more practice with word problems. How many apples will you need to trade?

Team D: You have a group of students who are excelling in math. They need to challenge themselves. How many apples will you need to trade?
45 minute classes
Option 1 - 45 minutes

30 students, 4 groups

- Monday - whole group lesson, mini-lesson, student-centered work session
- Tuesday - small group rotations
- Wednesday - small group rotations
- Thursday - Formative assessment, whole group lesson
- Friday - Common formative assessment

60 minutes classes
Option 2 - 60 minutes

30+ students, 4 groups

- Monday - whole group, mini-lesson, student-centered, independent practice
- Tuesday - small group rotation
- Wednesday - small group rotation
- Thursday - Formative assessment, whole group lesson
- Friday - Common formative assessment

http://ccgpsmathematics9-10.wikispaces.com/
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